Economic Growth and Development: Coursework Assignment 1

Matěj Adámek

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University of Plymouth

Student: 10531306

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Assignment description

- Section A: you should answer all questions.
- Section B: you should answer one question of your choice.
- Please report all the steps involved in the solution of each question. Reporting only a final result will be considered a partial answer.
- If a question/sub-question is accompanied by an asterisk (*), that means you have to underpin your arguments with appropriate referenced sources.
- Word limits (excluding diagrams, tables, and equations/maths) are indicated for questions (or sub-questions) that require some explanations or comments. Where a word limit is indicated, you have to report the word count. Penalty for each word count not reported: 0.5 point.
- Diagrams and mathematical derivations/calculations can be hand-written, but otherwise the answers must be word processed.
- Points for each question are indicated in the text. Total points sum to 100.

Section A

Question 1

$Y_i = K_i^{\frac{1}{2}} L_i^{\frac{1}{2}};\ \delta_A = \delta_B = 0.05;\ \gamma_A = 0.15;\ \gamma_B = 0.1.$

a: Per-worker production function of an economy is obtained by dividing both sides of the aggregate production function $Y_i = K_i^{\frac{1}{2}} L_i^{\frac{1}{2}}$ by the total number of workers L_i, as shown in equation (1). Given that output per worker $\frac{Y}{L} \equiv y$ and capital per worker $\frac{K}{L} \equiv k$, the expression may be rewritten in form shown by expression (2).

$$\frac{Y_i}{L_i} = \frac{K_i^{\frac{1}{2}} L_i^{\frac{1}{2}}}{L_i} = \frac{K_i^{\frac{1}{2}}}{L_i^{\frac{1}{2}}} = \left(\frac{K_i}{L_i}\right)^{\frac{1}{2}} \tag{1}$$

$$y_i = k_i^{1/2} \therefore \underline{\boldsymbol{y_A = k_A^{1/2}}} \land \underline{\boldsymbol{y_B = k_B^{1/2}}} \tag{2}$$

b: Steady state equilibrium is a point on the production function, at which investment per worker equals depreciation of capital per worker, that is $\boldsymbol{\gamma_i y_i = \delta_i k_i}$. (Weil 2016)

c: Within the steady state equilibrium condition $\gamma_i y_i = \delta_i k_i$, y_i may be substituted with the per-worker production function derived in equation (3). The resulting relationship, expressed in terms of k_i (4), can be evaluated to return value of steady state capital stock k_i^*.

Then, the per-worker production function (5) shall be used to determine the steady state equilibrium output corresponding to k_i^*.

Finally, steady state level of consumption c_i^*, representing standard of living, is equal to the proportion of output per worker that is not invested in capital. Mathematically, this is represented in equation (6). Evaluated, all three of these relationships are shown in table 1 for countries A and B.

$$\gamma_i y_i = \delta_i k_i \leftrightarrow \gamma_i k_i^{1/2} = \delta_i k_i \tag{3}$$

$$k_i^{1/2} = \frac{\gamma_i}{\delta} \leftrightarrow \underline{\boldsymbol{k_i^* = \left(\frac{\gamma_i}{\delta}\right)^2}} \tag{4}$$

$$\boldsymbol{y_i^* = k_i^{1/2}} \tag{5}$$

$$\boldsymbol{c_i^* = (1-\gamma_i) y_i^*} \tag{6}$$

i	A	B
k_i^*	$\left(\frac{0.15}{0.05}\right)^2 = 3^2 = \underline{9}$	$\left(\frac{0.10}{0.05}\right)^2 = 2^2 = \underline{4}$
y_i^*	$9^{1/2} = \underline{3}$	$4^{1/2} = \underline{2}$
c_i^*	$0.85 \times 3 = \underline{2.55}$	$0.9 \times 2 = \underline{1.8}$

Table 1: Steady State Equilibrium values

d: The Solow Growth model assumes investment in capital to be equal to savings, which in turn depend on consumption behavior of the population. This may be among other factors affected by the rate of return on investment, level of education in the population, short-term vs. long-term orientation of the society, or degree of property right enforcement.

In case of this example, Country A may be experiencing greater rate of investment due to higher interest rates on savings, providing the population with higher returns. In order to match country A, government of country B may choose to imply a policy forgoing tax on interest earned by one individual up to a certain value. Such measure would reduce the degree of interference with market forces in the banking industry, while increasing the net interest earned from private savings, hence providing an incentive for increase in the savings rate. [146/250]

Question 2

Given the same production function and depreciation rate as in Question 1, while setting $y \leq 3 \Rightarrow \gamma = 0.15 \ \wedge y > 3 \Rightarrow \gamma = 0.3$, steady state equilibrium capital stock value for both conditions can be found utilizing the relationship derived in previous question, shown in equation (7). The steady state output per worker is then calculated using the corresponding per-worker production function (8).

$$k_i^* = \left(\frac{\gamma_i}{\delta}\right)^2 \tag{7}$$

$$y_i^* = k_i^{1/2} \tag{8}$$

i	$y \leq 3 \Rightarrow \gamma = 0.15$	$y > 3 \Rightarrow \gamma = 0.30$
k_i^*	$\left(\frac{0.15}{0.05}\right)^2 = 3^2 = \underline{9}$	$\left(\frac{0.30}{0.05}\right)^2 = 6^2 = \underline{36}$
y_i^*	$9^{1/2} = \underline{3}$	$36^{1/2} = \underline{6}$

Table 2: Steady State values assuming $\gamma(y)$

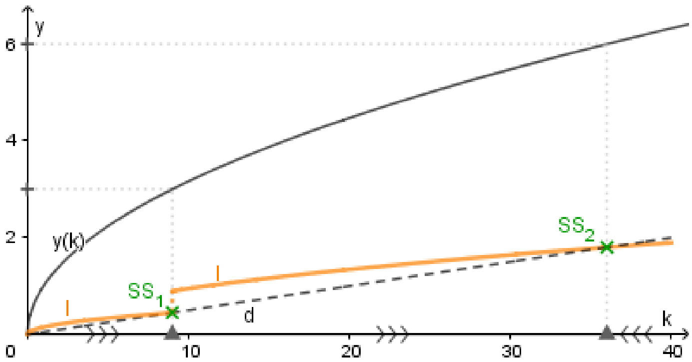


Figure 1: Illustration of question 2

For any initial income $y > 3$, the investment rate $\gamma = 0.3$, which corresponds to the steady state equilibrium output per worker $y^* = 6$ and steady state value of capital stock $k^* = 36$.

Should the economy however start at $y \leq 3$, y will converge towards value of 3 (y^*) over time, never exceeding the threshold for increase in interest rate. Because of this, such economy is in a poverty trap, unable continue growing to its potential.

Question 3

a: The cultural change induces upward shift in population growth rate curve from n_1 to n_2 in figure 2. In the short-run, population growth rate n increases to level of n_{sr}, while output per worker remains unchanged. Over time, labor force L grows at rate n, causing movement to the left along the relationship L_1 between population and output per worker. This results in decline in y, and a progressive decrease in population growth rate n. In the long run, n returns to 0 (steady state), while output per worker stabilizes at y_{ss2}.
$n \uparrow \Rightarrow L_{ss} \uparrow \Rightarrow y_{ss} \downarrow \Rightarrow n \downarrow$ [79/200]

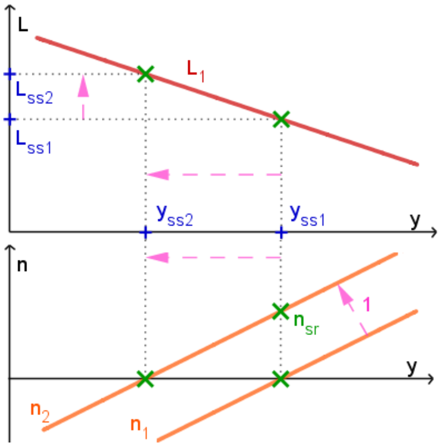


Figure 2: Long run effect of fertility change

b: Country A has higher productivity, hence higher steady state income per capita y_{ss} at each number of workers L, i.e. its labor-output function therefore lies above that of country B. However, because both countries exhibit equal population growth rate function, their steady state ($n = 0$) output per worker will be equal, $y_{ssA} = y_{ssB}$. The difference between production functions L_A and L_B will consequently translate into $L_{ssA} > L_{ssB}$. [57/200]

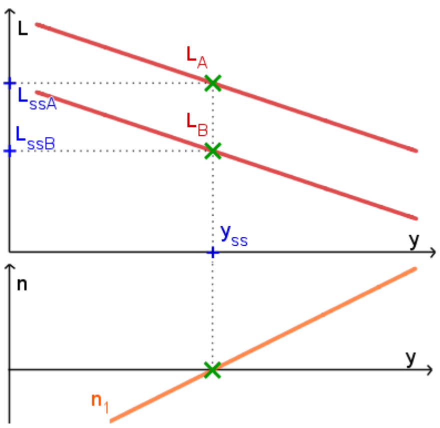


Figure 3: Result of productivity difference

Question 4*

Assuming that growth rate is approximated by logarithmic transformation of a variable (Koop 2008), the cobb-douglas production function (9) shall be adjusted to express relationships between growth rates of output $\hat{y}$, productivity $\hat{A}$, physical capital $\hat{k}$, and human capital stock $\hat{h}$. Equation, (10) expresses this in terms of the unknown variable $\hat{A}$. (Weil 2016, pp.192-193)

$$y_t = A_t k_t^{\alpha} h_t^{1-\alpha} \tag{9}$$

$$\hat{A} = \hat{y} - \alpha\hat{k} - (1-\alpha)\hat{h} \tag{10}$$

Then, the provided information may be used to calculate the known average annual growth rates using formula (11), as done in equations (12)(13), and (12).

$$\hat{x} = \left(\frac{x_{t+n}}{x_t}\right)^{\frac{1}{n}} - 1 \tag{11}$$

$$\hat{y} = \left(\frac{9}{3}\right)^{\frac{1}{65}} - 1 = 1.70\% \tag{12}$$

$$\hat{k} = \left(\frac{16}{4}\right)^{\frac{1}{65}} - 1 = 2.16\% \tag{13}$$

$$\hat{h} = \left(\frac{6}{3}\right)^{\frac{1}{65}} - 1 = 1.07\% \tag{14}$$

Using this information, the average annual growth rate of productivity $\hat{A}$ can be written in a form of linear function (16) of one unknown, that is α.

$$\hat{A} = 0.0170 - \alpha 0.0216 - (1-\alpha)0.0107 \tag{15}$$

$$\hat{A} = 0.0063 - \alpha 0.0109 \tag{16}$$

Because the model assumes constant positive returns to scale, the exponent $\alpha \in \langle 0,1 \rangle$ (Weil 2016). Average annual growth rate of productivity between 1950 and 2015 $\hat{A}$ must then fall within interval $\langle -0.40\%, 0.63\% \rangle$ as per expression **(17)**. Individual $\hat{A}$ corresponding to key values of α are summarized in table 3.

$$\hat{A} = 0.0063 - \langle 0,1 \rangle 0.0109 = \langle 0.63\%; -0.40\% \rangle \tag{17}$$

Finally, the contribution productivity growth to the total output growth between 1950 and 2015 y_A is calculated using total growth function (18), while assuming both production factors k and h constant, and hence their respective growth rates $\hat{k}$ and $\hat{h}$ equal to 0. Altered by these assumptions, the function (19) is evaluated in expressions (20) and (21). The results imply that the growth rate of productivity contributed between $\langle -0.75, 1.50 \rangle$ to the total output growth over the period y_A, depending on the value of α. Additional values of y_A for different levels of α are also shown in table 3.

$$y_{2015} = y_{1950} \times \hat{y}^{65} = y_{1950}(\hat{A} + \alpha\hat{k} + (1-\alpha)\hat{h})^{65} \tag{18}$$

$$y_A = y_{1950} \times \hat{A}^{65} \tag{19}$$

$$y_A = y_{1950} \times \langle -0.40\%, 0.63\% \rangle^{65} \tag{20}$$

$$y_A = \langle -0.75, 1.50 \rangle \tag{21}$$

α	$\hat{A}$	y_A
0	0.63%	1.500
1/3	0.27%	0.573
1/2	0.09%	0.183
7/12	0.00%	0.003
2/3	-0.09%	-0.165
1	-0.40%	-0.75
↑	↓	↓

Table 3: $\hat{A}$, and y_A for key values of α

[210/200]

Question 5*

The production function given is $y_i = A_i k_i^{\alpha} h_i^{1-\alpha}$, where i represents index of given country, A is the produtivity coefficient, k represents physical capital accummulated, h refers to human capital stock, and y to the output per worker in given country. Taking into account $\alpha = 1/3$, the production function may be rewritten as $y_i = A_i k_i^{1/3} h_i^{2/3}$.

Because the values provided are expressed relatively to a benchmark country $i = 0$, as y_i/y_0, the appropriate relationship shall be expressed as in equation (22).

$$\frac{y_i}{y_0} = \frac{A_i}{A_0}\left(\frac{k_i}{k_0}\right)^{1/3}\left(\frac{h_i}{h_0}\right)^{2/3} \tag{22}$$

In order to calculate relative factor accummulation F_i/F_0, relevant expression from the relative ouptut formula (22) shall be used. This is expressed in equation (23) with the division property of indices spplied.

$$\frac{F_i}{F_0} = \left(\frac{k_i}{k_0}\right)^{1/3}\left(\frac{h_i}{h_0}\right)^{2/3} \tag{23}$$

The original relative output function (22) substituted with F_i/F_0 can then be expressed in terms of relative productivity A_i/A_0 as in equation (24) and evaluated using available values. The results of both calculations are shown in table 4.

$$\frac{y_i}{y_0} = \frac{A_i}{A_0}\frac{F_i}{F_0} \Leftrightarrow \frac{A_i}{A_0} = \frac{y_i}{y_0}\Big/\frac{F_i}{F_0} \tag{24}$$

i	y_i/y_0	k_i/k_0	h_i/h_0	F_i/F_0	A_i/A_0
A	0.80	0.78	0.74	$0.78^{\frac{1}{3}}0.74^{\frac{2}{3}} = 0.753$	$0.80/0.753 = 1.062$
B	0.10	0.30	0.40	$0.30^{\frac{1}{3}}0.40^{\frac{2}{3}} = 0.363$	$0.10/0.363 = 0.275$
0	1	1	1	1	1

Table 4: Relative F and A

In case of country A, the relatively lower output per worker to benchmark country is exclusively caused by lower factor accummulation. Country B on the other hand exhibits both lower factor accumulation than the benchmark country.

[174/200]

Question 6*

According to Lorentzen et al. (2008), "greater risk of death during the prime productive years is associated with [...] lower investment in physical capital," which suggests, that a policy that reduces mortality is expected to have a positive impact on physical capital accumulation over time. With respect to magnitude of this effect, they find statistically significant that increase in adult mortality by 10 percentage points induces 1.93 percentage point decrease in proportion of GDP spent on investment. [73/150]

Question 7*

Technological progress relies primarily on willingness of private firms to innovate. Innovation is however risky, which results in low expected returns, and almost none in case given invention can be easily imitated by competition through observation. (Carlin & Soskice 2014, Weil 2016)

For this reason, government may choose to implement patent policy and allow innovators to each monopolistic rents. Despite this measure results in reduced degree of competition in concerned markets, the contribution of innovation may outweigh the consequential decrease in total welfare.

Nevertheless, the inverted quadratic relationship between degree of innovation and market competition observed by Aghion et al. (2005) implies that some degree of competition must be preserved in order to facilitate further innovation, as firms with very high degree of market power protected by patents tend to innovate less. That is why, patents are generally granted for a limited period of time only. [136/200]

Section B

Question 8

Investment rate γ; depreciation rate δ; population growth rate $n = 0$; Steady state equilibrium; ouptut per worker y; capital per worker k; productivity constant A; $y = Ak^{\alpha}$ $\alpha \in (0,1)$; Consider permanent $\uparrow \gamma$

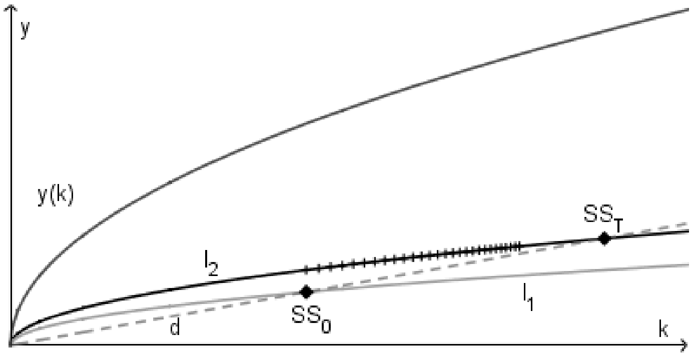


Figure 4: Effect of $\gamma \uparrow$ on k_{ss}

[63/120]

a: Steady state equilibrium is defined as a point where $\gamma y = \delta k$, i.e. proportion of y invested equals proportion of capital depreciated. Substituting in the production function $y = Ak^{\alpha}$, this can be expressed in terms of k as in equation (26). Assuming $\alpha, \delta \in (0,1)$, $\gamma \uparrow \implies k_{ss} \uparrow$.

$$\gamma y_{ss} = \delta k_{ss} \leftrightarrow \gamma A k_{ss}^{\alpha} = \delta k_{ss} \leftrightarrow k_{ss}^{1-\alpha} = \frac{\gamma A}{\delta} \leftrightarrow \tag{25}$$

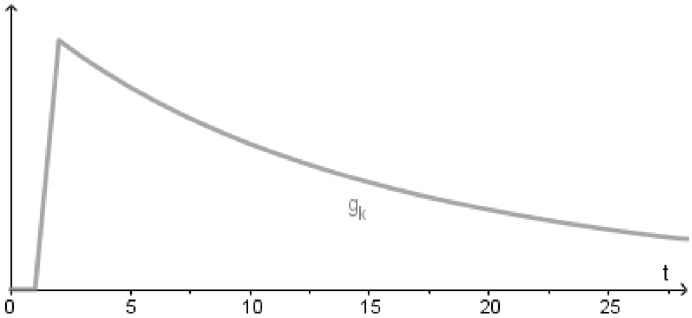
$$\leftrightarrow k_{ss} = \left(\frac{\gamma A}{\delta}\right)^{\frac{1}{1-\alpha}} \tag{26}$$

Growth rate of capital stock $\hat{k}$ is equal to residual investment after correcting for depreciation divided by current level of capital stock k_t, as shown in equation (27). The initial $\gamma \uparrow$ will result in $\hat{k}_1 \uparrow$, assuming $A, k \in R^{+} \wedge \alpha \in (0,1)$.

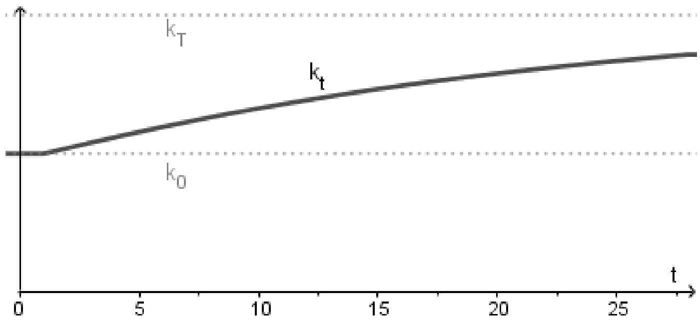
$$\hat{k}_t = \frac{\gamma y_t - \delta k_t}{k_t} = \frac{\gamma A k_t^{\alpha} - \delta k_t}{k_t} = \gamma A k_t^{\alpha - 1} - \delta \tag{27}$$

$$k_{t+1} = k_t \hat{k}_t \tag{28}$$

b: Following the increase in γ and consequential spike in growth rate of capital stock $\hat{k}$, will be its convergence toward 0. Capital stock k_i will therefore commence to increase by diminishing increments each year.



(a) $\hat{k}$



(b) k

Figure 5: Time series plot

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Written with LyX, images generated in GeoGebra.