

Financial Economics Course Work 1

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1 Design and analysis

(1.1) Constant growing annuity is calculated as the difference between two constant growth perpetuities(Brealey et al. 2012). My desired annual cashflow C is £24,000 increasing every year at growth rate g of 2.5% for a total period t of 20 years. The annual discount rate r is 5%.

$$t = 20; g = 0.025; r = 0.05; C = 24000$$

$$\begin{aligned} PV &= \frac{C}{r-g} - \frac{C \times (1+g)^t}{r-g} \frac{1}{(1+r)^t} \\ &= 24000 + \frac{24000}{0.05 - 0.025} - \frac{24000 \times (1 + 0.025)^{20}}{0.05 - 0.025} \frac{1}{(1 + 0.05)^{20}} = \\ &= 24000 + 960000 - \frac{39327}{0.025 \times 1.05^{20}} = \underline{\underline{\text{£391,125.79}}} \end{aligned}$$

By the time I retire, I will need to have saved £391,125.79.

(1.2) Chosen property: <http://www.rightmove.co.uk/property-for-sale/property-44647068.html>

The price of the property p is £1.3M. I will have to pay £130k upfront, taking out a PV £1,170,000 mortgage. The bank offers a t 30-year mortgage at an annual interest rate r of 4% compounded monthly, which means that the monthly interest rate $\frac{r}{m}$ is 0.33% and the total number of monthly payments $t \times m$ is 360.

$$p = 1300000; C_0 = 0.1p = 130000; PV = 1170000$$

$$r = 4\%; m = 12; \frac{r}{m} = 0.33\%; t = 30; t \times m = 360$$

$$C = ?$$

$$PV = C \left(\frac{1}{\frac{r}{m}} - \frac{1}{\frac{r}{m}} \times \frac{1}{(1 + \frac{r}{m})^{t \times m}} \right)$$

$$1170000 = C \left(\frac{1}{0.0033} - \frac{1}{0.0033} \times \frac{1}{1.0033^{360}} \right) = C \left(300 - \frac{300}{3.313} \right) = 209.448C$$

$$C = \frac{1170000}{209.448} = \underline{\underline{\text{£5,587}}}$$

In order to repay the mortgage without getting repossessed, I will have to pay £5,587 every month for 30 years (Brealey et al. 2012).

(1.3) Question three compares the present value, duration, and volatility of two identical bonds with different compounding. Face value, or principal, C_p was set at £1000, the coupon rate $\frac{C_i}{C_p}$ at 8%, bond's time to maturity t is 30 years, and finally its yield to maturity r at 3%. Part 1.3.2 then assumes semi-annual compounding.

1.3.1 Present value of a bond is calculated as shown below (Brealey et al. 2012).

$$C_p = 1000; \frac{C_i}{C_p} = 8\%; C_i = 80$$

$$t = 30; r = \frac{0+6}{2} = 3\%$$

$$PV_{bond} = PV_{annuity} + PV_{principal}$$

$$PV_{bond} = C_i \left(\frac{1}{r} - \frac{1}{r} \frac{1}{(1+r)^t} \right) + \frac{C_p}{(1+r)^t}$$

$$PV_{bond} = 80 \left(\frac{1}{0.03} - \frac{1}{0.03} \frac{1}{(1.03)^{30}} \right) + \frac{1000}{(1.03)^{30}}$$

$$PV_{bond} = 80 \times 19.6 + 411.987 = \underline{\underline{\text{£1,979.987}}}$$

The the present value, or price, of the bond is £1,979.987. In order to calculate duration of the bond, following formula is used (Brealey et al. 2012). Microsoft Office Excel was used to evaluate the summative equation.

$$\text{Duration} = \sum_{i=1}^{N-1} \left(\frac{i \times PV(C_i)}{PV_{bond}} \right) + \frac{N \times PV(C_p + C_N)}{PV_{bond}}$$

$$\text{Duration} = \sum_{i=1}^{29} \left(\frac{i \times \frac{80}{1.03^i}}{1979.987} \right) + \frac{30 \times \frac{1080}{1.03^{30}}}{1979.987}$$

$$\text{Duration} = 10.04 + 6.74 = \underline{\underline{16.78 \text{ years}}}$$

The invested principal of the bond is repaid in 16 years, 9 months, 14 days, 21 hours, 28 minutes and 48 seconds.

Modified duration, which expresses the volatility of given bond, is then calculated as $MD = \frac{\text{Duration}}{1+r}$ (Brealey et al. 2012). The result below shows that with an increase of 1% in the interest rate, the price of the bond decreases by 16.29%.

$$MD = \frac{16.78}{1.03} = \underline{\underline{16.29\%}}$$

1.3.2 Supposing a change from annual to semi-annual compounding, the same indicators are calculated and compared.

$$C_p = 1000; \frac{C_i}{C_p} = 0.08; C_i = 40$$

$$t = 30; r = \frac{0+6}{2} = 3\%; m = 2$$

$$PV_{bond} = \frac{C_i}{m} \left(\frac{1}{\frac{r}{m}} - \frac{1}{\frac{r}{m}} \frac{1}{(1 + \frac{r}{m})^{t \times m}} \right) + \frac{C_p}{(1 + \frac{r}{m})^{t \times m}}$$

$$PV_{bond} = 40 \left(\frac{1}{0.015} - \frac{1}{0.015} \frac{1}{(1.015)^{60}} \right) + \frac{1000}{(1.015)^{60}}$$

$$PV_{bond} = 40 \times 39.38 + 409.296 = \underline{\underline{\text{£1,984.496}}}$$

The present value, or price, of the bond is £1,984.496. Therefore, it is valued at almost £5 higher price.

$$\text{Duration} = \sum_{i=1}^{N-1} \left(\frac{i \times PV(C_i)}{PV_{bond}} \right) + \frac{N \times PV(C_p + C_N)}{PV_{bond}}$$

$$\text{Duration} = \frac{\sum_{i=1}^{59} \left(\frac{i \times \frac{40}{1.015^i}}{1984.496} \right) + \frac{60 \times \frac{1040}{1.015^{60}}}{1984.496}}{2}$$

$$\text{Duration} = \frac{20.71 + 12.87}{2} = \underline{\underline{16.78 \text{ years}}}$$

The invested principal of the bond is repaid at approximately the same time as with annual compounding.

$$MD = \frac{16.78}{1.03} = \underline{\underline{16.29\%}}$$

The volatility of the semi-annually compounded bond is essentially the same as with annual compounding. The conclusion therefore is that it is better to invest in the annually compounded bond, which saves me money to buy a coffee while I wait for my coupon payments.

(1.4) The dividend currently paid D_0 was £5. Future dividends are expected to grow at a growth rate g_a of 5% a year for 5 years, then at g_b of 3% for 3 more years and then at g_c of 2% indefinitely. The required rate of return r is equal to the average of the last two digits of my student number, i.e. $\frac{0+6}{2} = 3\%$.

$$D_0 = \text{£}5; r = 3\%; H = 8; g_a = 5\%; g_b = 3\%; g_c = 2\%$$

$$PV = \sum_{t=1}^H \left(\frac{D_t}{(1+r)^t} \right) + \frac{D_H}{r - g_{H+1}} \frac{1}{(1+r)^H}$$

t	1	2	3	4	5	6	7	8	H+
g_t	5%	5%	5%	5%	5%	3%	3%	3%	2%
$D_t = D_{t-1}(1 + g_t)$	5.25	5.51	5.79	6.08	6.38	6.57	6.77	6.97	$\frac{6.97}{0.03 - 0.02} = 697.31$
$PV(D_t) = \frac{D_t}{(1+r)^t}$	5.10	5.20	5.30	5.40	5.50	5.50	5.50	5.50	$\frac{697}{(1.03)^8} = 550.47$
$\Sigma PV(D_t)$	43.01								-
P_0	<u>593.47</u>								

Table 1: Process of calculation of current value P_0 of the stock

As calculated above, using formulae from Brealey et al. (2012), the stock is currently valued at £593.47.

(1.5) Dividend just paid D_0 was £8. The expected growth rate of future dividends g is 3% a year, while required rate of return r is 10%. The company pays out 55% of its profits to the investors, reinvesting the remaining 45%.

$$D_0 = 8; g = 3\%; r = 10\%; \text{Payout ratio} = 55\%$$

$$RoE = \frac{g}{1 - \text{Payout ratio}} = \frac{0.08}{1 - 0.55} = 17.78\%$$

$$PVGO = 0.1778 - 0.10 = \underline{\underline{7.78\%}}$$

$$PV = \frac{D_0}{r - g} = \frac{8}{0.10 - 0.03} = \text{£}114.29$$

Present value of growth opportunities, or PVGO, determines the relationship between company growth and stock price over time. PVGO is the net present value of firm's future investments and is calculated as $PVGO = RoE - r$, where RoE is the return on equity and r is the required rate of return. RoE is then calculated as $RoE = \frac{\text{Earnings per share}}{\text{Book equity per share}}$ or alternatively as $RoE = \frac{g}{1 - \text{Payout ratio}}$ (Brealey et al. 2012).

If $PVGO > 0$, the price of the stock will increase over time. Therefore, with the data introduced above, this company's stock price will grow in the future.

(1.6) Chosen devices:

<https://www.o2.co.uk/shop/phones/alcatel/20.45x/#contractType=payasyougo>
<https://www.carphonewarehouse.com/nokia/150.html#!colour=white&dealType=pg>
<http://shop.vodafone.co.uk/shop/pay-as-you-go/alcatel-2051-payg/sku93589>

The annual discount rate r determined by my student number is 3%.

$$EAC = \frac{P \times r}{1 - (1 + r)^t}$$

Retailer	Three	Carphone Warehouse	Vodafone
Model	Alcatel 2045	Nokia 150	Alcatel 2051
Price P	£27.99	£19.99	£22.50
Lifespan t	4 years	5 years	3 years
EAC	£6.69	£3.77	£7.28

Table 2: Comparison of expected annual cost of devices

The phone with lowest expected annual cost is Nokia 150 from Carphone Warehouse. It is therefore the best option of the three (Brealey et al. 2012).

(1.7) Using the econometric software GRETL, standard deviation σ of each stock's returns can be obtained and the variances σ^2 calculated. The same program computes their correlation $\rho_{1;2}$. (Cottrell & Lucchetti 2011)

$$\sigma_A = 5.9918 \implies \sigma_A^2 = 35.9017; \sigma_B = 6.1840 \implies \sigma_B^2 = 38.2419; \rho_{A;B} = 0.3737$$

Portfolio variance $Pvar = \sum_{j=1}^N \sum_{k=1}^N w_j w_k \rho_{j;k} \sigma_j \sigma_k$, where w is the proportion of the portfolio formed by given stock, while N the total number of stocks represented. $N \equiv 2 \rightarrow Pvar = (w_A^2 \sigma_A^2) + (w_B^2 \sigma_B^2) + 2(w_A w_B \rho_{A;B} \sigma_A \sigma_B)$ (Brealey et al. 2012).

$$\therefore Pvar = 35.9017w_A^2 + 38.2419w_B^2 + (2 \times 0.3737 \times 5.9918 \times 6.1840)w_A w_B \\ 27.6936$$

Weights for minimum variance portfolio can be found by minimizing the portfolio variance function subject to $w_1 = 1 - w_2$, i.e. the condition that the sum of weights of both stocks must be equal to 1.

$$\min: Pvar(w_A; w_B) \text{ s.t. } w_B = 1 - w_A$$

$$w_B = 35.9017w_A^2 + 38.2419(1 - w_A)^2 + 27.6936w_A(1 - w_A) = 46.45w_A^2 - 48.79w_A + 38.24$$

$$\frac{dw_A}{dw_A} = 92.9w_A - 48.79 \therefore 92.9w_A - 48.79 \implies w_A = \underline{\underline{0.5252}} \wedge w_B = 1 - w_A = 1 - 0.5252 = \underline{\underline{0.4748}}$$

The minimum variance portfolio consists of 52.52% of stock A and 47.48% of stock B.

(1.8) A β of a stock is calculated as shown below. Therein, $\sigma_{i;m}$ is the covariance of stock's returns to the market portfolio return, while σ_i^2 is the variance of the returns of the stock.

$$\beta_i = \frac{\sigma_{i;m}}{\sigma_i^2}$$

$$\sigma_A^2 = 35.9017; \sigma_B^2 = 38.2419; \sigma_{A;m} = 4.59; \sigma_{B;m} = 24.27$$

$$\beta_A = \frac{4.59}{35.9017} = \underline{\underline{0.1278}}$$

$$\beta_B = \frac{24.27}{38.2419} = \underline{\underline{0.6346}}$$

Beta represents the sensitivity of stock's returns to changes in return of the market portfolio. The results indicate that β of both stocks is lower than 1, suggesting that they are less volatile than the market portfolio. Effectively, both of the stocks present with lower risk than the market portfolio but provide a lower expected return.

(1.9) Using the CAPM model, the values of β for each of the two stocks can be calculated (Brealey et al. 2012).

$$r_f = 3\%; r_m = 6.5\%; \beta_A = 0.1278; \beta_B = 0.6346$$

$$r_i = r_f + \beta_i(r_m - r_f)$$

$$r_A = 0.03 + 0.1278(0.065 - 0.03) = \underline{\underline{3.45\%}}$$

$$r_B = 0.03 + 0.6346(0.065 - 0.03) = \underline{\underline{5.22\%}}$$

(1.10) The following graph represents the CAPM model of this market with stocks A and B shown. All stocks below the red line are overvalued, while all above it are undervalued. This is due to the fact that higher β calls for compensation of the risk in form of higher r . The market portfolio and risk-free rate show two key alternatives. If a stock is overvalued, market portfolio, or risk-free investment shall be therefore purchased instead (Brealey et al. 2012).

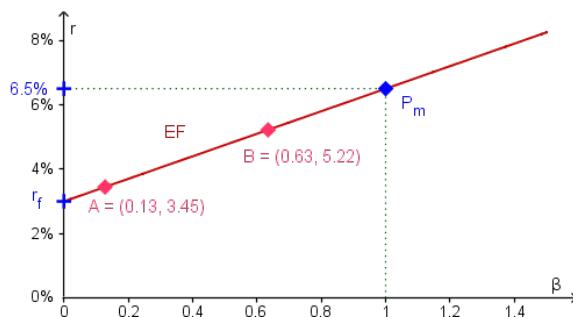


Figure 1: CAPM efficient frontier

(1.11) Standard deviation σ is a type of descriptive statistic that expresses the average deviation of values from their mean. In other terms, it represents the absolute volatility of stock's returns over time. $\sigma = \sqrt{\frac{1}{N} \sum_{i=1}^N (x_i - \bar{x})^2}$, where N is the number of evaluated periods, x_i the value of returns in each period, and \bar{x} is the arithmetic mean of all variables over time. The standard deviation is expressed in units of the variable that is being analyzed (Koop & Quinlivan 2000).

However, because investors organize their portfolios within a certain market, there is need for indicator β expressing volatility relative to the market portfolio, i.e. a portfolio consisting of all the securities in given market, returns of which are the market returns. β is calculated as

$$\beta_x = \frac{\sigma_{x;y}}{\sigma_x^2} = \frac{\sum_{i=1}^N (x_i - \bar{x})(y_i - \bar{y})}{\sum_{i=1}^N (x_i - \bar{x})^2}$$

where x represents returns of analyzed stock, while y represents the market return. Upper bar signifies average, while i in lower index suggests a value of variable in period i (Brealey et al. 2012, LeRoy & Werner 2014, Koop & Quinlivan 2000).

The main difference between these two indicators is that β accounts for market volatility, while σ does not. If the stock returns are highly volatile, but their covariance to the market returns is low, suggesting that their variation is not explained by changes in the market returns, β of the stock is going to be low (Brealey et al. 2012).

(1.12) Certainty equivalent cashflow CEQ_t is dependent on investor's expected utility of income, i.e. their risk-attitude (Perloff 2013). The Most investors are risk-averse, which is why $EV(C_t) < C_t$. Given that the expected risky cashflow in year t 7, C_7 is £300, risk-free rate r_f 3%, the market portfolio rate of return r_m 5%, and the beta of assessed stock β_p is 1.5, the CEQ_7 can be calculated as shown below (Brealey et al. 2012).

$$C_t = £300; r_f = 3\%; r_m = 5\%; \beta_p = 1.5; t = 7$$

$$r_p = r_f + \beta_p(r_m - r_f) = 0.03 + 1.5(0.05 - 0.03) = 6\%$$

$$PV = \frac{C_t}{(1+r_p)^t} = \frac{CEQ_t}{(1+r_f)^t}$$

$$\frac{300}{1.06^7} = \frac{CEQ_7}{1.03^7} \implies CEQ_7 = \frac{300}{1.06^7} \times 1.03^7 = \underline{\underline{£245.38}}$$

(1.13) The Net present value of a project is calculated as a sum of its cashflows C_t discounted at an interest rate r in each year (Brealey et al. 2012). Cashflow for every year is calculated as $C_t = \text{Profit}_t - \text{Cost}_t - I_t$, where I_t represents investment into the project in given year. Based on this, the year at which costs exceed profits can be determined in advance, which is why the project will be terminated at its beginning. Corresponding costs and profits, which are dependent on variable growth rates are shown in table 3. In the last row therein, the sum of discounted cashflows, i.e. the net present values, is calculated.

$$r = 5\%; \text{Cost}_1 = 80; \text{Profit}_1 = 100; I_1 = 50$$

$$\text{NPV} = \sum_{t=0}^N \frac{C_t}{(1+r)^t}$$

t	Cost _t	Profit _t	I _t	C _t	$\frac{C_t}{1.05^t}$
0	0	0	50	-50	-50
1	80	100	0	20	19
2	88	100	0	12	10.9
3	95	106	0	11	9.5
4	102.6	112.4	0	9.8	8.0
5	110.9	119.1	0	8.2	6.5
6	119.7	126.2	0	6.5	4.9
7	129.3	133.8	0	4.5	3.2
8	139.6	141.9	0	2.3	1.5
9	150.8	150.4	-	-	-
Σ					<u>£13.43</u>

Table 3: Project 1.13 cashflows

(1.14) Assuming that the price PV_1 of a perpetual stock is £50 and the rate of return r is 14%, its dividend D_1 can be calculated. Then, using the CAPM model, β_1 is found based on r , the risk free rate r_f and the market risk premium ($r_m - r_f$). Should β_1 change by $\% \Delta \beta$ 3% to β_2 , the new rate of return r_2 can be determined and used to find the new price PV_2 .

$$\% \Delta \beta = -0.03; r = 14\%; r_f = 7\%; (r_m - r_f) = 9\%$$

$$PV_1 = D_1/r_1 \implies 50 = D_1/0.14 \implies D_1 = £7$$

$$r_1 = r_f + \beta_1(r_m - r_f) \implies 0.14 = 0.07 + \beta_1 \times 0.09 \implies \beta_1 = 0.778$$

$$\beta_2 = (1 + \% \Delta \beta) \beta_1 = 0.97 \times 0.778 = 0.754$$

$$r_2 = r_f + \beta_2(r_m - r_f) = 0.07 + 0.754(0.09) = 0.1379$$

$$PV_2 = D_1/r_2 = 7/0.1379 = **£50.76**$$

(1.15) Company's cost of capital, or rate of return on assets, is calculated as a weighted average of rates of return on its liabilities (Brealey et al. 2012). The values of their equity E, as well as their debt D is known and can be summed to represent their total value V. The rate of return on debt is assumed to be the same as for the treasury bill $r_f = r_D$, i.e. 2%. Then, From their stock's β , the market risk premium ($r_m - r_f$), and the risk free rate r_f , the rate of return on their equity shall be calculated. Finally, the weighted average of these two shows that the given company's cost of capital is 7%.

$$E = £20M; D = £5M \quad \beta = 1.25; \quad r_f = r_D = 2\%; \quad (r_m - r_f) = 5\%$$

$$V = E + D = £25M$$

$$r_E = r_f + \beta(r_m - r_f) = 0.02 + 1.25(0.05) = 8.25\%$$

$$COC = r_D \left(\frac{D}{V}\right) + r_E \left(\frac{E}{V}\right) = 0.02 \left(\frac{5}{25}\right) + 0.0825 \left(\frac{20}{25}\right) = 0.004 + 0.066 = 7\%$$

2 Concept understanding

(2.1) The small firm effect, an empirically observed market anomaly identified by Banz (1981), associates low market capitalization in firms with higher average returns. In literature, multiple reasons for this occurrence are presented (Banz 1981).

Firstly, a firm with a small market capitalization may have more growth opportunities than large firms, which gives them a chance to use their financial capital more efficiently. Secondly, he argues that some areas in small businesses may be underfunded, which is corrected for by increasing flow of funds from the financial market, yielding a higher return compared to large companies. Finally, the price of small firm's stocks may be lower due to higher initially perceived risk. The volatility of the stock returns decreases over time, as the firm grows, yielding higher returns, while the value of its β decreases.

Lustig & Leinbach (1983) argue that this is not necessarily an inefficiency in the market, as such effect may be observed and adjusted for. What it does imply, according to them, is a misspecification in the Capital Asset Pricing model. The CAPM had been under a large wave of criticism since the late 1970s, which is why Fama & French (1992) have worked to include more effects of publicly available variables on stock's rate of return, working under the semi-strong form of efficient market hypothesis.

van Dijk (2011) however points out that there is literature claiming that this effect has disappeared in the late 1980s. He assesses the current state and argues that although size effect still exists, none of its evidenced causes have a solid base on economic theory. He calls for further research, stressing that Banz (1981) himself admitted that the size effect may only be a "proxy for one or more true unknown factors."

273 words

(2.2) The statement “If you put people in a lab experiment, they’ll behave the way you expect them to, not the way they’d do naturally” was introduced by List (2004), while he inspected the contrasts between the neoclassical utility of income theory and the more modern prospect theory, which was mostly based on a closed-world laboratory experiments. The latter stresses that agents will rather retain their current endowment, rather than trading it for an object of uncertain value. This behavior results from what is referred to as the endowment effect (Perloff 2013).

List (2004) conducted an experiment based on more open-world ontology, observing a number of agents with various levels of experience in marketplace trade. Its results have shown a disparity between the participants: The experienced traders (marketplace dealers) have been less reluctant to proceed to exchange their endowment for alternative good, while the regular market customers more. This implied that the former group of individuals tends to act more in accordance with the neoclassical theory of utility of income, while the latter adhered to the prospect theory.

In financial markets, the situation is deemed to comply to the results of this research. List (2004), Levitt & List (2007) both claim that in real world, the magnitude of endowment effect is likely to change based on current setting and value of options relative to their personal income.

Furthermore, Levitt & List (2007) points out that in a laboratory experiment the agent, being aware of the setting, will likely behave in a different way than in the real world. Also, the initial endowments and payoffs provided in a laboratory experiment can hardly ever reach values similar to the financial market situations. In a laboratory experiment, the difference between loss and gain is therefore not high enough for the individual to care about. Financial market movements, however have the potential to influence personal well-being of a trader. Additionally, depending on their confidence, the traders may perceive risks taken with each transaction to be lower in relation to something-or-nothing situation, which often occurs among these experiments.

328 words

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