Intertemporal Choice

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University of Plymouth

Student: 10531306

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Essay Question

A consumer is considering her consumption choices of a particular good across two periods. Using relevant diagrams and equations, explain how the consumer makes her choice. Discuss the factor(s) that affect the consumer's choice and the effects of changes in this/ these factor(s) on the consumer's welfare. Outline the assumptions that govern your analysis.

Similar to consumer choice, intertemporal choice is a concept related to the topic of consumer behavior analysis. Unlike in case of consumer choice, however, the examined agent decides between consumption of “an arbitrary good now or at a given point if future” (Varian 2006), rather than consumption of two different goods at one point in time. This essay examines the mathematical framework used for analysis of intertemporal choice, which was first equipped by Dr. Irving Fisher. (Thaler 1997)

To begin with, it is necessary to state assumptions, under which such model is applicable. Firstly, the assumption of perfect certainty is established, as values of all variables need to be exact. (Hite 1999) Secondly, capital markets are assumed to be perfect, i.e. there is one market interest rate, same for both saving and borrowing. (Hey 2003) Then, the concept of ordinal utility is necessary; utility function of the consumer is known and the utility of consumption bundle is quantifiable in utils.(Block & Wutscher 2014) Finally, this framework represents a two-period world, where only two periods occur, therefore all income is spent. (Varian 2006) The main aim of this model is to find an affordable consumption bundle with highest utility under these circumstances.

Being aware of the prerequisites of the model, variables it uses and their respective symbols need to be presented. The endogenous variables, values of which are found by the model are quantities of a good consumed in periods 1 and 2, c_1 and c_2 respectively. For each consumption bundle $CB = [c_1, c_2]$, there is a certain level of utility U defined by utility function $U(c_1, c_2)$.(Block & Wutscher 2014) Exogenous variables necessary to find an intertemporal budget constraint are levels of income m_t and prices of the given good p_t in each of the two periods t. Additionally, real interest rate r determines the interest on both savings and borrowings between the two periods. Furthermore, rate of inflation π represents the change in price level, i.e. the relationship between p_1 and p_2, which is $p_2 = p_1(1+\pi)$. Finally, the effects of the last two variables introduced can be aggregated into real interest rate $(1+\rho) = \frac{(1+r)}{(1+\pi)} \implies \rho = \frac{r-\pi}{(1+r)}$. (Hey 2003, Varian 2006)

In situations, where intertemporal choice is considered, the first step is finding set of all highest affordable consumption bundles. Represented on a graph, where c_1 is on horizontal axis and c_2 on vertical, this is represented by a linear function called budget constraint.

Given endowment of m_1 and m_2, the consumer has an option to reallocate their income between the two periods as demonstrated in figure 1a. Firstly, they can increase their c_1 by borrowing in period 1 and then repaying the amount in period 2. The maximum amount of money they can borrow is entire m_2 discounted by interest paid for the loan, i.e. the present value of m_2, which can be mathematically denoted as $PV(m_2) = \frac{m_2}{(1+r)}$. Analogically, if they choose to maximize their c_2, they have option to save the entire endowment of m_1, gaining interest on it and increasing their spending in period 2. This is known as the future value of m_2 and can be calculated as

$FV(m_1) = m_1(1+r)$. (Hey 2003) Present and future value can also be calculated for any c_2 and c_1 respectively. Assuming that $p_1 = p_2 = 1$, $PV(c_2) = \frac{c_2}{(1+r)}$, and $PV(c_1) = c_1 \times (1+r)$.

Based on this and the assumption of that all money is spent by the end of period 2, it is possible to establish that, expressed in terms of the same period, the sum of income must be equal to the cost of consumption. Hence, setting period 1 as reference, this can be mathematically denoted as in equation (1). Similarly, equation (2) expresses the same relationship, considering period 2 as a reference. Holding the values of c_1 and c_2 equal to 0, equation (1) will represent the intercept of the budget constraint with c_1 axis, while equation (2) the intercept with c_2 axis. However for purposes of further mathematical analysis, it is more efficient to deal with its functional form, expressed, as in equation (3) (Varian 2006), in terms of the variable c_2.

$$c_1 + PV(c_2) = m_1 + PV(m_2) \implies c_1 + \frac{c_2}{(1+r)} = m_1 + \frac{m_2}{(1+r)} \tag{1}$$

$$FV(c_1) + c_2 = FV(m_1) + m_2 \implies c_1(1+r) + c_2 = m_1(1+r) + m_2 \tag{2}$$

$$c(c_1) = m_2 + (1+r)m_1 - (1+r)c_1 \tag{3}$$

In order to include the effect of prices into the budget constraint, the quantities consumed need to be multiplied by their respective prices. Applied on the future value form of budget constraint, this is shown in equation (4). Expressed in terms of c_2, this would look as equation (5). (Varian 2006) Furthermore, it is possible to simplify this formula, p_2 can be substituted with $p_1(1+\pi)$ as in equation (6). Finally, equation (7) is the general formula of intertemporal budget constraint IBC.

$$c_1(1+r) \times p_1 + c_2 \times p_2 = m_1(1+r) + m_2 \tag{4}$$

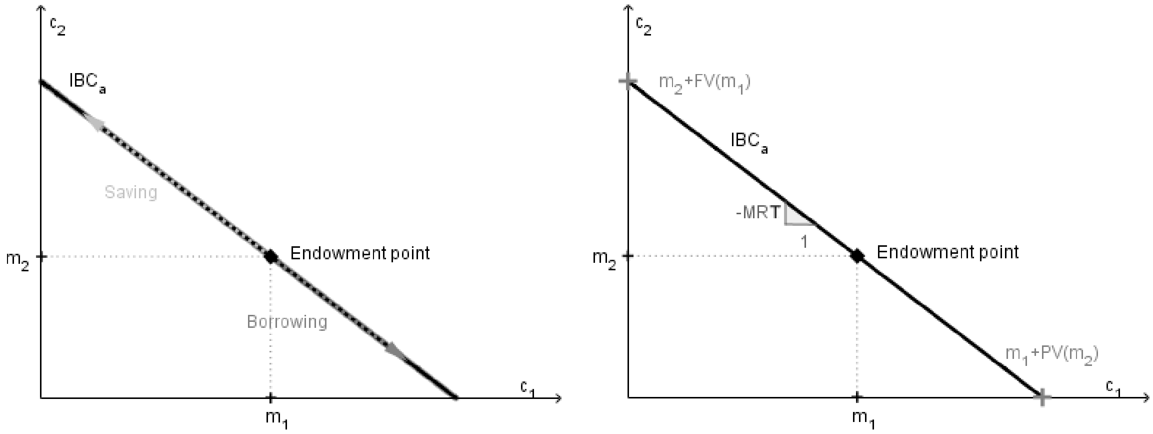
$$c_2(c_1) = \frac{m_1(1+r)}{p_2} + \frac{m_2}{p_2} - \frac{c_1(1+r)p_1}{p_2} \tag{5}$$

$$c_2(c_1) = \frac{m_1(1+r)}{p_1(1+\pi)} + \frac{m_2}{p_1(1+\pi)} - \frac{c_1(1+r)p_1}{p_1(1+\pi)} \tag{6}$$

$$c_2(c_1) = \frac{m_1(1+r)}{p_1(1+\pi)} + \frac{m_2}{p_1(1+\pi)} - c_1\frac{(1+r)}{(1+\pi)} \tag{7}$$

The slope of the budget constraint represents effect of increasing period 2 consumption by marginal unit on m_1. In order to find slope, the first derivative of the function needs to be expressed as shown in equation (8). Diagram in figure 1b shows this graphically and explains that the negative of the slope is called marginal rate of transformation (MRT) and represents the cost of trade-offs.

$$\frac{dc_2}{dc_1} = -\frac{(1+r)}{(1+\pi)} = -(1+\rho) = -MRT \quad (8)$$



(a) Saving and borrowing (b) Intercepts and marginal rate of transformation

Figure 1: Diagrams illustrating intertemporal budget constraint

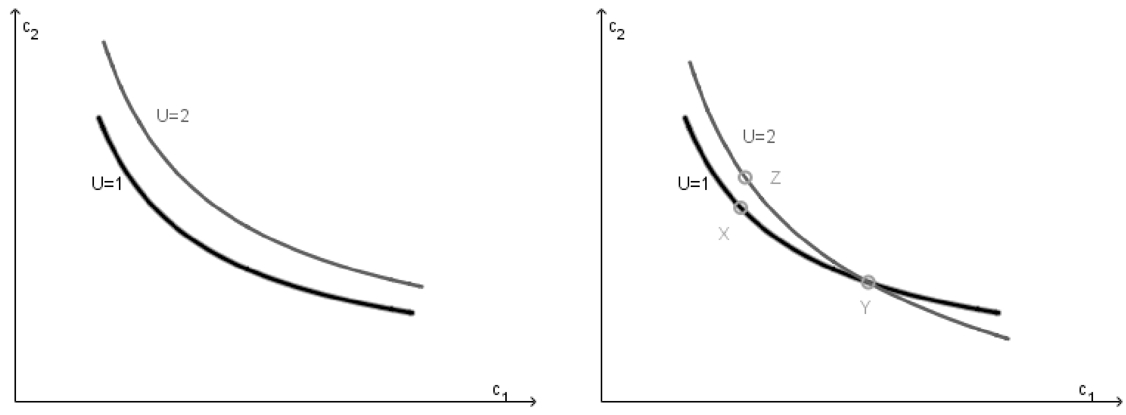
At this point, all the highest affordable pairs of consumption now and consumption later are known, therefore the first condition, i.e. finding all optimal consumption bundles, has been fulfilled. However, the aim of the model is finding such bundle that is also utility maximizing. For that reason, information about consumer's individual preferences between the two options is necessary.

Because utility is presumed to be ordinal in nature, these can be expressed in form of a utility function $U(c_{1,}c_2)$. For each possible pair of c_1 and c_2, it returns total utility expressed in the unit of utils. In theory, there are three basic types of utility functions, each of which represents a different type of relationship between the two periods of consumption, as shown in table 1 (Varian 2006).

Relationship of c_1 & c_2	$U(c_1,c_2)$	Indifference curve	MRS
Perfect substitutes	$U = ac_1 + bc_2$	$c_2 = \frac{\bar{U}-ac_1}{b}$	$\frac{dc_2}{dc_1} = -\frac{a}{b}$
Imperfect (Cobb-Douglas)	$U = c_1^{\alpha}c_2^{\beta}$	$c_2 = (\frac{\bar{U}}{c_1^{\alpha}})^{\frac{1}{\beta}}$	$\frac{MU(c_1)}{MU(c_2)} = \frac{\alpha c_2}{\beta c_1}$
Perfect complements	$U = min(c_1;c_2)$	L-shaped	0

Table 1: 3 basic types of utility function

In order to deal with the utility function graphically in a two-dimensional model, it must be represented as a set of indifference curves. An indifference curve is a projection of the two-variable utility function at a fixed level of utility, i.e.$\bar{U}(c_1,c_2)$. In order to plot it on the graph, it is necessary to reorder the formula and express it in terms of c_2. For each of the three basic cases of utility functions, the third column of table 1 shows corresponding indifference curve.



(a) 2 sample indifference curves of a Cobb-Douglas production function

(b) Proof of transitivity by contradiction

Figure 2: Indifference curve plots

The three main properties of indifference curves include completeness, transitivity, and monotonicity. Firstly, completeness meaning that for any value of c_1, there is a corresponding value of c_2. Secondly, Monotonicity implies that “more is better”, i.e. if at any point, c_1 is increased by 1, the new consumption bundle must be above the current indifference curve. This implies negative first derivative and hence downward slope. (Hite 1999) Then, Transitivity suggests that indifference curves derived from one utility functions never cross. This can be proven by contradiction, given 3 arbitrary consumption bundles X,Y,Z. If $X \sim Y$, i.e. both are points on the same indifference curve and $Z \succ X \therefore Z \succ Y$, Z is presumably on a higher indifference curve. If, however, two indifference curves were to cross at point Y, suggesting that $Y \sim Z \succ X \sim Y$, this assumption would be violated. The negative of the slope of an indifference curve is known as the marginal rate of substitution MRS, i.e. the amount of c_2 that has to be given up in order to gain 1 extra unit of c_1, at a fixed level of utility.

Returning to the original aim, which is to find an affordable consumption bundle with highest level of utility, it is now possible to find such point, as details about both intertemporal budget constraint and intertemporal utility function resp. its indifference curves are known. In mathematical terms, the primary intent is to maximize utility, given a budget constraint, as denoted in equation (9). Furthermore, for better illustration in the two-dimensional model, the same problem can be rephrased as finding a tangent point between the budget constraint and highest achievable indifference curve. For such consumption bundle, it is true that $MRT = MRS$.

For Cobb-Douglass production function, this problem is solved in substituted with values shown in equations (10), (11), and (12). Firstly, MRS is substituted in and MRT rewritten as the value from table 1, then expressed in terms of c_1 in equation (10). Then, in equation (11), the result is substituted into the general formula of intertemporal budget constraint from equation (7) and evaluated in order to find the numerical value of the equilibrium value c_2^*. Lastly, corresponding c_1^* is found in equation (12) by simply evaluating the budget constraint formula reordered in terms of c_1, as all the other values are known at this point.

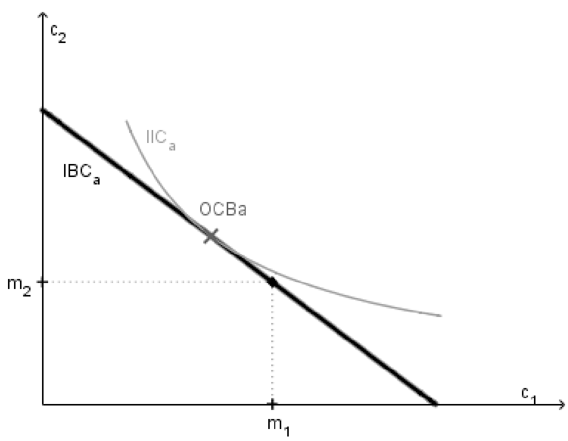
The optimal utility-maximizing consumption bundle is then found at point $OCB = [c_1^*, c_2^*]$. Using this point, the saving behavior of the agent can be determined as seen in figure 3. If $c_1^* < \frac{m_1}{p_1}$, i.e. if their OCB is towards northwest from the initial endowment point, they are a saver. In contrast, they are a borrower, if $c_1^* > \frac{m_1}{p_1}$.

$$\begin{aligned} &\underset{c_1,c_2}{maximize} \quad U(c_1,c_2) \\ &subject\ to \qquad c_2(c_1) \end{aligned} \tag{9}$$

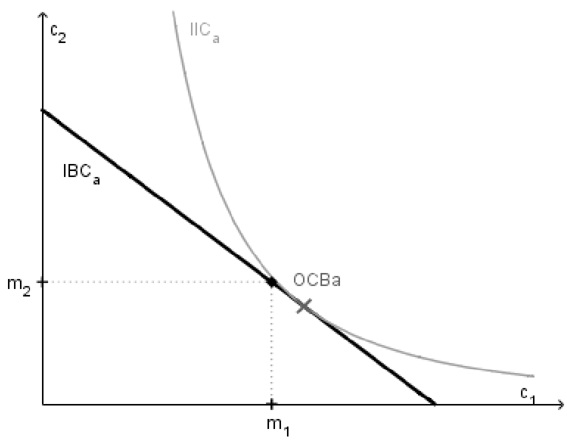
$$MRT = MRS \implies \frac{(1+r)}{(1+\pi)} = \frac{\alpha c_2}{\beta c_1} \implies c_1 = \frac{\alpha c_2(1+\pi)}{\beta(1+r)} \tag{10}$$

$$c_2^* = \frac{m_1(1+r)}{p_1(1+\pi)} + \frac{m_2}{p_1(1+\pi)} - \frac{\alpha c_2^*(1+\pi)(1+r)}{\beta(1+r)(1+\pi)} \implies c_2^* = \frac{\beta(\frac{m_1(1+r)}{p_1(1+\pi)} + \frac{m_2}{p_1(1+\pi)})}{\beta+\alpha} \tag{11}$$

$$c_2^* = \frac{m_1(1+r)}{p_1(1+\pi)} + \frac{m_2}{p_1(1+\pi)} - c_1^*\frac{(1+r)}{(1+\pi)} \implies c_1^* = \frac{(1+\pi)(\frac{m_1(1+r)}{p_1(1+\pi)} + \frac{m_2}{p_1(1+\pi)} - c_2^*)}{(1+r)} \tag{12}$$



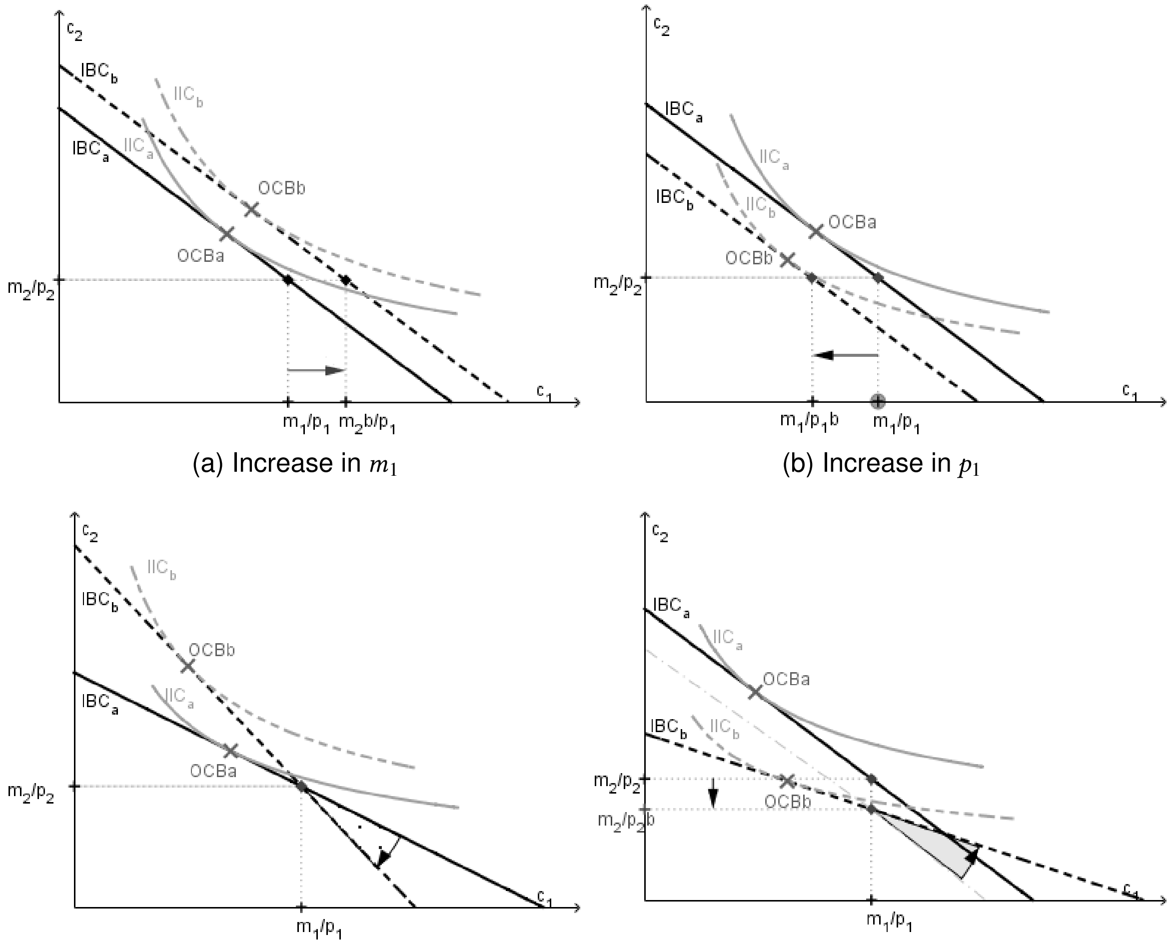
(a) Saver



(b) Borrower

Figure 3: Graphical representation of example optimal consumption bundle

In order to assess effects of changes in each of the variables in the model and determine their nature, it is necessary to relate back to the variables this model uses. In table 2, each of the relevant variables is shown and its effect characterized. Additionally, figure 4 presents all of these effects graphically, in the same order. (Hey 2003) The resulting effect on welfare is observed by the change of the fixt utility level the indifference curve passing through the new optimal consumption bundle repreaents.



(a) Increase in m_1

(b) Increase in p_1

(c) Increase in r

(d) Increase in π

Figure 4: Illustration of result of changes in exogenous variables, considering a saver.

Variable	Effect of increase in variable	Effect on welfare	
		saver	borrower
m_t	Intercepts ↑	↑	
p_1	Intercepts ↓	↓	
r	Slope ↓ (steeper)	↑	↓
π	c_2 intercept ↓ Slope ↑ (flatter)	↓	ambiguous

Table 2: Exogenous variables and their effect on the model

Each of these changes aggregates two effect of two influences, an income effect and substitution effect. Income effect determines how much of the change between two situation occurred as a result of decrease in purchasing power of individuals income. Substitution effect, then then quantifies the change caused by the nature of consumer's utility function. These two are visualized in (5). (Perloff 2013) The mathematical relationship is then characterized by the Slutzky equation (13) (Varian 2006).

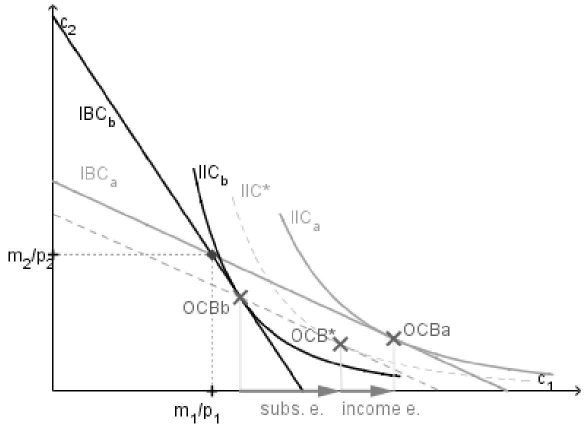


Figure 5: Slutzky approach to graphical decomposition of total effect

$$\frac{\Delta c_1^{total}}{\Delta p_1} = \frac{\Delta c_1^{substitution}}{\Delta p_1} + (m_1 - c_1)\frac{\Delta c_1^{income}}{\Delta m} \tag{13}$$

In conslusion, intertemporal choice is the instrument used by agents who decide between consumption of a good in current and following period. The mathematical model used to evaluate this builds on the model of consumer choice and includes relevant additional variables. Aim of the consumer is to choose from all efficient consumption bundles represented by a budget constraint such that the selected consumption bundle maximizes their utility given by consumer-specific utility function.

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Diagrams generated using GeoGebra 5.0.295

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